Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Optimal Catapult Impulse Shaping for Ejection Seats

P.K.A. Menon* and R.A. Walker†
Integrated Systems Inc.
Palo Alto, California

Introduction

THE importance of the crew escape concepts in advanced aircraft development is well documented for both practical and humanitarian reasons. The objective is to control an ejection seat system to significantly reduce injuries over a larger aircraft operating envelope. Control technologies are fundamental to this objective. Acceptable accelerations and rotational rates must be maintained to avoid collisions and unnecessary impulsive loads from wind blast and parachute openings.

Jines[1] has described the standard Air Force ejection seat as a manned vehicle, suggesting that modern control theory is a promising tool to yield a vectored thrust digital flight controller that will improve performance while maintaining human tolerance limits. Carroll[2] describes initial research in methodologies to achieve such a vectored thrust control law during the post-catapult phase. Higgins[3] has given a detailed analysis of the ejection seat catapult dynamics in a high-acceleration environment.

In this Note, attention will be focused on the problem of optimal open-loop acceleration input shaping during the catapult phase to maximize the delivered impulse while maintaining the crewperson dynamic response index within low risk limits. A linear biodynamical model due to Brinkley and Shaffer[4] is used in this analysis. One of the main conclusions that emerged out of this study is that an optimally scheduled catapult impulse can significantly reduce the risk of serious injury due to acceleration stresses.

This Note will not deal with any trajectory analysis, but will assume that the impulse required to achieve adequate ejection speeds to avoid interference with aircraft extremities has been computed. The remaining task then is the selection of catapult acceleration shape to minimize the crew person dynamic response index (DRI) while delivering the required impulse.

Optimal Control Problem

The spinal injury model from Brinkley and Shaffer[4] with a first-order lag rocket actuator model in state variable form is given below. The first-order lag is introduced to ensure the inclusion of a realistic rise time for the actuator.

$$\dot{x}_1 = x_2 \tag{1}$$

$$\dot{x}_2 = x_3\omega_n^2 - 2\delta\omega_n x_2 - \omega_n^2 x_1 \tag{2}$$

$$\dot{x}_3 = \tau_c[T - x_3] \tag{3}$$

where x_1 is the spinal deflection (in.), and T the commanded thrust (g's), the control variable. It is constrained as

$$0 \leq T \leq T_{\max}$$

where τ_c is the thruster time constant; δ the damping ratio, 0.224; and ω_n the natural frequency (rad/s), 52.9.

The effect of lateral accelerations is not considered here. However, the approach presented can be extended for these cases as well.

Assuming that the failure of the vertebral column can be related to the deflection in the model, the dynamic response index is computed as

$$\mathrm{DRI} = x_1(\omega_n^2/g)$$

There is a small ambiguity in the definition of DRI given in Ref. 4 and the way it is referred to in Fig. 11 of Ref. 1. The interpretation in this Note follows that of Ref. 1. A chart relating DRI and the spinal injury rate given in Fig. 4 of Ref. 4 will be used to evaluate the present work.

It is assumed here that the rocket catapult impulse per unit mass (ft/s) and its duration have been specified from trajectory considerations, i.e.,

$$\int_{t_0}^{t_f} x_3 \mathrm{d}t = C \qquad C \text{ is a given constant} \tag{4}$$

Since negative thrust is not permissible, it is clear that a given catapult impulse would produce a positive DRI history; and it

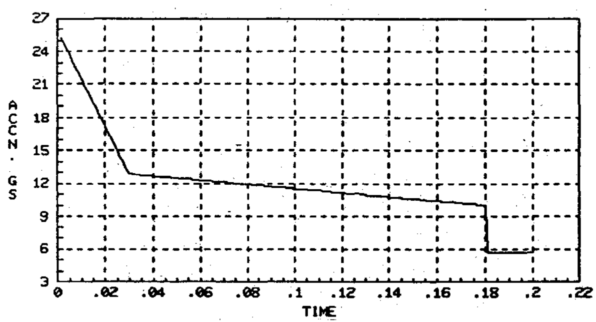

Fig. 1 F-111 catapult acceleration history at 450 keas.[4]

Table 1 Onset acceleration shape comparison

	Delivered impulse per unit mass, ft/s	$\mathrm{DRI}_{\max}$	% Probability of injury
Previous test acceleration shape	77.8	24	90
Optimal acceleration shape	78.9	13	0.2

Received July 19, 1984; presented as Paper 84-1895 at AIAA Guidance and Control Conference, Seattle, Wash., Aug. 20-22, 1984; revision received Oct. 25, 1984.

*Research Scientist. Member AIAA.
†Manager, Aircraft Systems. Member AIAA.

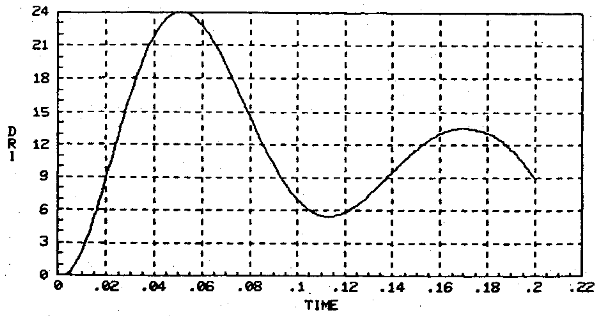

Fig. 2 Vertical (DRI) response to the F-111 catapult acceleration.

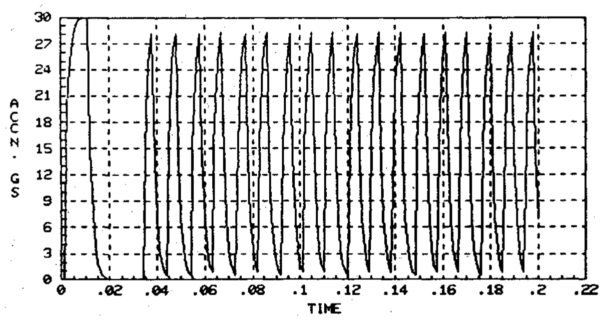

Fig. 3 Optimal pulsing acceleration time history.

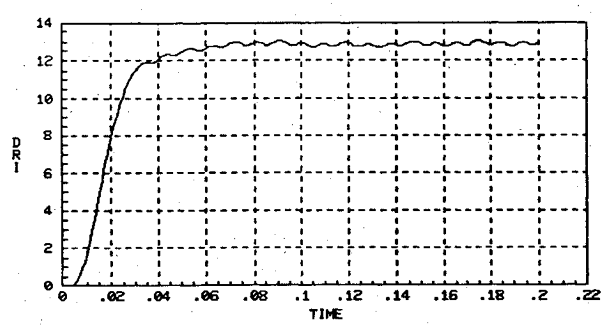

Fig. 4 Optimal DRI response for desired specific impulse level.

is the authors' intention to maintain this at a safe level, say DRI_s.

The optimal control problem is

$$\min_{T} \frac{1}{2} \int_{t_0}^{t_f} (\mathrm{DRI}_s - \mathrm{DRI})^2 \mathrm{d}t$$

subject to the state equations [Eqs. (1-3)] and the isoperimetric constraint [Eq. (4)].

The initial conditions $x_1(t_0)=0$, $x_2(t_0)=0$, $x_3(t_0)=0$ specified, and the final conditions $x_1(t_f)$, $x_2(t_f)$, and $x_3(t_f)$ are free. The variational Hamiltonian[5] can be formed as

$$H = \tfrac{1}{2}\left\{\mathrm{DRI}_s - \frac{\omega_n^2}{g} x_1\right\}^2 + \lambda_{x_1} \cdot x_2$$

$$+ \lambda_{x_2}[x_3\omega_n^2 - 2\delta\omega_n x_2 - \omega_n^2 x_1] + \lambda_{x_3}\tau_c[T - x_3] + \mu \cdot x_3$$

The Euler-Lagrange equations are

$$\dot{\lambda}_{x_1} = \left\{\mathrm{DRI}_s - \frac{\omega_n^2}{g} x_1\right\}\frac{\omega_n^2}{g} + \lambda_{x_2}\omega_n^2, \qquad \lambda_x(t_f) = 0 \tag{5}$$

$$\dot{\lambda}_{x_2} = -\lambda_{x_1} + \lambda_{x_2} 2\delta\omega_n, \qquad \lambda_{x_2}(t_f) = 0 \tag{6}$$

$$\dot{\lambda}_{x_3} = -\lambda_{x_2}\omega_n^2 + \lambda_{x_3}\tau_c - \mu, \qquad \lambda_{x_3}(t_f) = 0 \tag{7}$$

$$\dot{\mu} = 0, \qquad \mu \text{ is a constant} \tag{8}$$

Since the control variable appears linearly in the Hamiltonian, Pontriagin's minimum principle[5] requires that the control variable be chosen at each instant such that

$$T = T_{\max}, \quad \text{if} \quad \lambda_{x_3} < 0$$

$$= 0, \quad \text{if} \quad \lambda_{x_3} > 0 \tag{9}$$

Hence the control is bang-bang. Note that a singular arc can arise if $\lambda_{x_3} \equiv 0$ over a finite interval of time.[6] This aspect, however, will not be pursued any further in this Note.

Equations (1-9) form the linear two-point boundary-value problem to be solved to obtain the optimal catapult acceleration history shape. This problem can be solved in several ways. The approach adopted in the present case is a version of the successive approximation technique.[7] Note at this point that if one desires a smooth catapult impulse shape, the above problem could be reformulated by adjoining the square of the control with appropriate weights to the integrand of the performance index.

Illustrative Example: The F-111 Catapult

In order to assess the benefits of using optimal catapult shaping, F-111 catapult data from Ref. 4 is used for comparison. In Fig. 1, a straightline segment approximation to the acceleration measured at the module's center of gravity of an F-111 catapult ejection test at 450 keas is shown. This corresponds to an impulse of 77.8 lbf-s. This acceleration is used as the forcing function for the spinal injury model with a dynamic response index history in Fig. 2 resulting. The maximum value of DRI is 24 and corresponds to approximately 90% probability of injury, extrapolating from the spinal injury rate curve of Brinkley and Shaffer.[4]

Next, optimal catapult impulse shaping is carried out to achieve approximately the same impulse. Figure 3 shows an optimal acceleration time history. The catapult acceleration is nearly bang-bang, and would be if a first-order lag in the actuator had not been modeled ($\tau = 2$ ms compared to the 10-ms pulsing rate). Note that increasing the actuator time constant will have the effect of not allowing the acceleration history to fall so far toward zero before rising with another pulse. The pulse rate is determined by the maximum acceleration magnitude the catapult can deliver, assumed to be 30 g's here.

Figure 4 shows the resulting optimal DRI response to the acceleration in Fig. 3 with a maximum DRI value of approximately 13, for a delivered impulse per unit of mass of 78.9 ft/s. The risk of spinal injury corresponding to this case is 0.2%. It may not be possible to realize this pulsing rate of 100 Hz, but the significant conclusion from our optimal input analysis is that multiple pulses with appropriate delay, even an appropriately scaled two-pulse train, will reduce the resulting DRI significantly. Table 1 summarizes the results.

Conclusions

An approach to optimal catapult impulse shaping for minimizing the risk of spinal injury to the ejectee while providing sufficient impulse to ensure satisfatory post-ejection trajectory was discussed. The approach employed uses a linear biodynamical model due to Brinkley and Shaffer.[4]

A numerical study with F-111 catapult data showed that the probability of spinal injury can be decreased significantly by employing the proposed approach. Further sophistication in the analysis is feasible using more complex biodynamical modes. Future research needs to address the effect of initial conditions at the ejection instant on the optimal catapult impulse shape. Near-optimal closed-loop mechanization also needs to be investigated.

References

[1]Jines, L.A., "The Air Force Ejection Seat as a Vehicle for Digital Flight Control," *Proceedings of the AIAA Guidance and Control Conference,* Gatlinburg, Tenn., Aug. 1983, pp.

[2]Carroll, J.V., "Control Law Design for Ejection Seats," *Proceedings of the AIAA Guidance and Control Conference,* Gatlinburg, Tenn., 1983, pp.

[3]Higgins, A.M., "Catapult Dynamics in a High Acceleration Environment," USAF Academy, Colo. USAFA-TN-82-5, Oct. 1982.

[4]Brinkley, J.W. and Shaffer, J.T., "Dynamic Simulation Techniques for the Design of Escape Systems: Current Applications and Furture Air Force Requirements," Symposium on Biodynamic Models and Their Applications, Wright-Patterson AFB, Ohio, AMRL-TR-71-29, Dec. 1971.

[5]Bryson, A.E. and Ho., Y.C., *Applied Optimal Control,* Hemisphere Publishing Co., Washington, D.C., 1975.

[6]Kelley, H.J., Kopp, R.E., and Moyer, G.H., "Singular Extremals," *Topics in Optimization,* edited by G. Leitmann, Academic Press, New York, 1967.

[7]Neustadt, L., "Synthesizing Time Optimal Controls," *Journal of Mathematical Analysis and Applications,* Vol. 1, 1960, p. 484.

Absolute Stability of Symmetric Highly Maneuverable Missiles

M. Guelman*
Rafael, Ministry of Defence, Haifa, Israel

Introduction

THE design of cruciform missiles has been based on the use of three independent channels for roll pitch and yaw control. This basic design is based on the assumption that the interaction among these channels is low. This assumption is valid if the system works at low angles of attack. When higher angles of attack are required for increased maneuverability, the coupling terms become significantly dominant and the channel independence assumption is no longer valid. A different approach is then required for flight control system design.

This basic need was already identified by different researchers in this field. In particular, Ref. 1 is worth mentioning, where, using the model in the performance index concept, a coupled auto-pilot control was defined. The main limitation of this interesting work is to be found in the model used for analysis. As the authors themselves indicate, nonlinear effects were neglected and only the roll yaw dynamics were considered. Also, the pitch channel was assumed to be uncoupled and independently controlled. In Ref. 2, the roll nonlinear effects were explicity considered, but pitch yaw dynamics were completely neglected.

A full understanding of the open loop system behavior is a basic requirement in control design before an attempt at synthesizing a control policy can be made.

As is well known,[3,4] the combined pitch yaw roll dynamics of an aerodynamically controlled missile is highly complicated. The system model is necessarily of high order and strongly nonlinear.

Received Aug. 9, 1984; revision received Nov. 19, 1984.

*Head, Department of Guidance and Control.

The purpose of the present work is to, first, define a system model, which is tractable in analytic terms, for a cruciform symmetric highly maneuverable missile. Next, based on this model, study the system behavior, and, in particular, find the conditions under which the open loop system is absolutely stable.

System Definition

In Fig. 1, a cruciform symmetric vehicle in atmospheric flight is depicted. The vehicle is aerodynamically controlled. In order to mathematically represent the angular motion of the vehicle at least eight state variables are required: p,q,r are the body rates in body coordinates; α,β, the angles of attack and sideslip, positive nose up and left, respectively, required to define the aerodynamic forces and moments; and ψ,θ,ϕ, the body angles with respect to inertial coordinates.

The differential equations relating these state variables are highly nonlinear. Realistic assumptions will be made in order to arrive at a system tractable in analytical terms. These assumptions are as follows:

1) Gravity effects are neglected. This assumption is valid for the highly maneuverable vehicles, here considered, capable of applying tens of g's in an arbitrary direction.

2) The moment of inertia matrix is of the form: $I_{ij}=0$ if $i\neq j$, $I_x \ll I_y = I_z$.

3) $\sin\alpha \sim \alpha$, $\cos\alpha \sim 1$; $\sin\beta \sim \beta$, $\cos\beta \sim 1$.

4) The aerodynamic pitch and yaw forces and moments in body axes are linear functions of α and β.

5) The control surface moments can be considered separately from body moments.

6) The control surfaces forces and the body aerodynamic side (out of maneuver plane) forces and moments can be neglected.[5]

With these assumptions the system equations for p,q,r,α,β, using the flight path axes[3,4] are

$$\dot{p} = L - L_p p + L_\delta \tag{1}$$

$$\dot{q} = -M_q q + M_\alpha \alpha + pr + M_\delta \tag{2}$$

$$\dot{r} = -N_r r - N_\beta \beta - qp + N_\delta \tag{3}$$

$$\dot{\alpha} = -Z_\alpha \alpha - \beta p - q \tag{4}$$

$$\dot{\beta} = -Y_\beta \beta + \alpha p + r \tag{5}$$

These five equations relating p,q,r,α,β, are independent of the three additional state variables ψ,θ,ϕ. Furthermore, the entire open loop system anagular dynamic behavior and stability is defined by Eqs. (1-5).

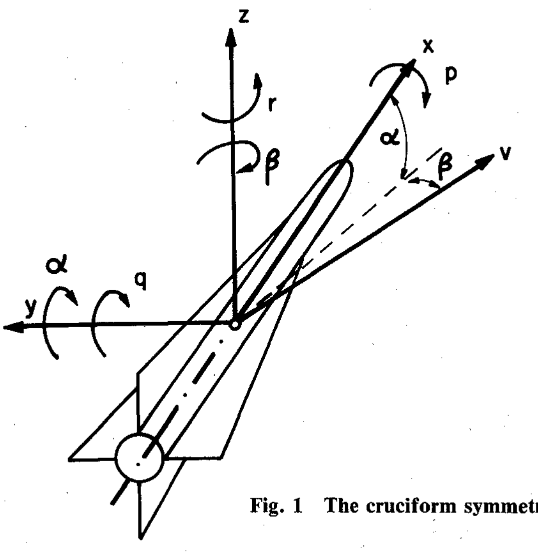

Fig. 1 The cruciform symmetric missile.